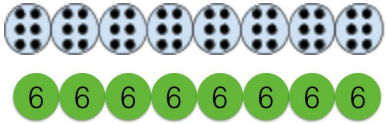
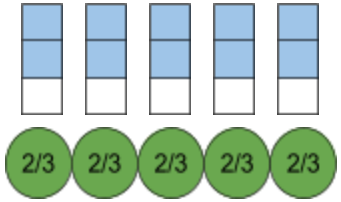
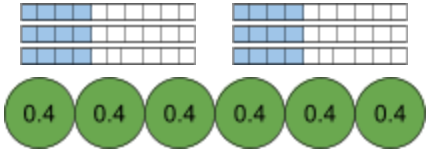
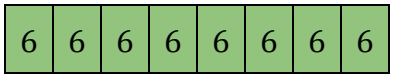
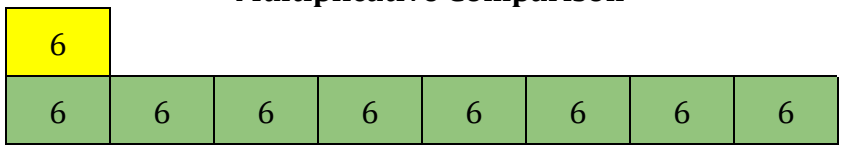
Visual Models:
Multiplication and Division
Grade 3 – Grade 5

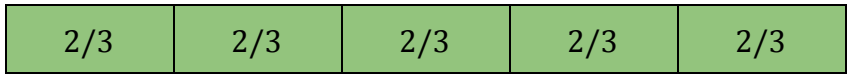
<table>
<tr><th></th><th>Grade 3</th><th>Grade 4</th><th>Grade 5</th><th>Grade 5</th></tr>
<tr><td>Equal Groups</td><td> 

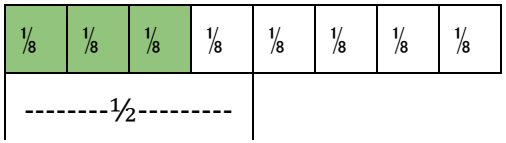
6+6+6+6+6+6+6+6 = 48
8 groups of 6 = 48
6 x 8 = 48 48 ÷ 6 = 8
8 x 6 = 48 48 ÷ 8 = 6</td><td> 

Fraction x Whole Number
$\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{10}{3}$
5 groups of $\frac{2}{3}=\frac{10}{3}$
$5 \times \frac{2}{3}=\frac{10}{3}$
$\frac{2}{3} \times 5=\frac{10}{3}$</td><td>Fraction ÷ Whole number
$\frac{10}{3} \div 5=\frac{2}{3}$
10/3 ÷ 2/3 = 5</td><td> 

0.4+0.4+0.4+0.4+0.4+0.4 = 2.4
6 groups of 0.4 = 2.4
6 x 0.4 = 2.4 2.4 ÷ 6 = 0.4
0.4 x 6 = 2.4 2.4 ÷ 0.4 = 6</td></tr>
<tr><td>Tape Diagram</td><td> 

6+6+6+6+6+6+6+6 = 48
6 x 8 = 48 48 ÷ 6 = 8
8 x 6 = 48 48 ÷ 8 = 6</td><td>Multiplicative Comparison
 

48 is 8 times as large as 6
6 x 8 = 48 48 ÷ 6 = 8
8 x 6 = 48 48 ÷ 8 = 6
 

Fraction x Whole Number
$5 \times \frac{2}{3}=\frac{10}{3}$
$\frac{2}{3} \times 5=\frac{10}{3}$</td><td>6 is ⅛ as large as 48
⅛ of 48 is 6
Fraction ÷ Whole Number
$\frac{10}{3} \div 5=\frac{2}{3}$
10/3 ÷ 2/3 = 5</td><td> 

$\frac{3}{4} \times \frac{1}{2}=\frac{3}{8}$ 3/8 ÷ 1/2 = 3/4
$\frac{1}{2} \times \frac{3}{4}=\frac{3}{8}$ 3/8 ÷ 3/4 = 1/2</td></tr>
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Grade 3 – Grade 5

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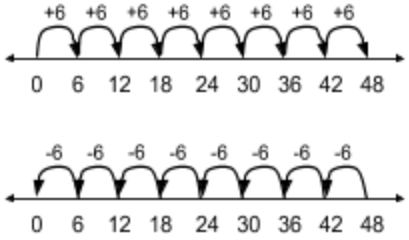
<tr><td>Open Number Line</td>
<td>


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48-6-6-6-6-6-6-6-6 = 0
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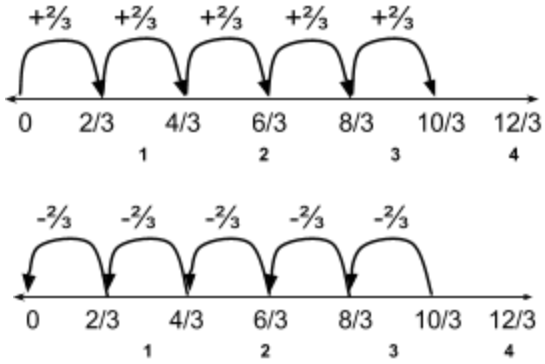
</td>
<td colspan="2">


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Fraction ÷ Whole Number
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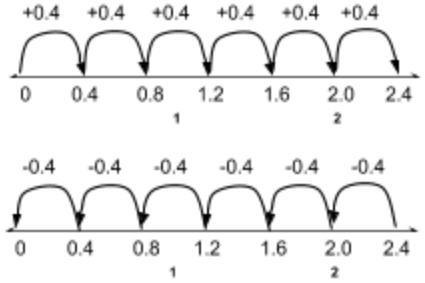

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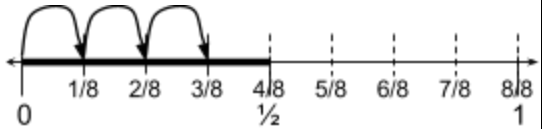

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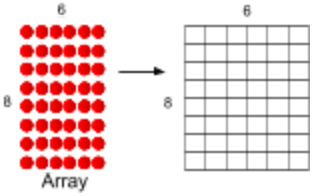
<tr><td rowspan="3">Area Model</td><td rowspan="2">Proportional</td><td> 

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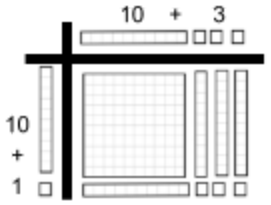
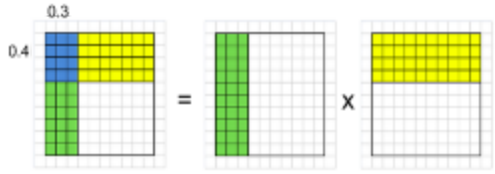
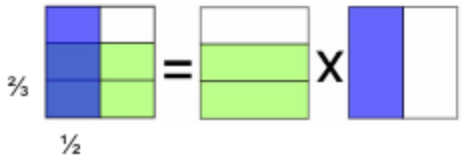
8 x 6 = 48 48 ÷ 8 = 6</td><td rowspan="2"> 

2-digit x 2-digit Whole Numbers
11 x 13 = 143
13 x 11 = 143</td><td rowspan="2">÷2-digit Divisor Whole Numbers
143 ÷ 13 = 11
143 ÷ 11 = 13</td><td> 

0.3 x 0.4 = 0.12 0.12 ÷ 0.3 = 0.4
0.4 x 0.3 = 0.12 0.12 ÷ 0.4 = 0.3</td></tr>
<tr><td> 

4 x 13 = 52 52 ÷ 4 = 13
13 x 4 = 52 52 ÷ 13 = 4</td><td> 

$\frac{2}{3} \times \frac{1}{2} = \frac{2}{6}$ 2/6 ÷ 1/2 = 2/3
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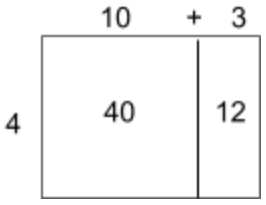
<tr><td>Generic</td><td> 

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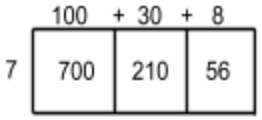
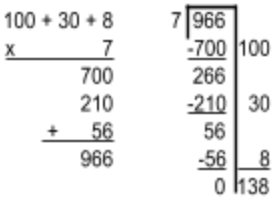
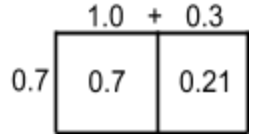
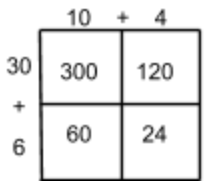
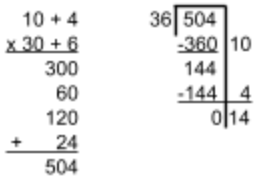
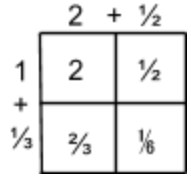
13 x 4 = 52 52 ÷ 13 = 4</td><td> 



7 x 138 = 966 966 ÷ 7 = 138
138 x 7 = 966 966 ÷ 138 = 7</td><td> 

0.7 x 1.3 = 0.91 0.91 ÷ 0.7 = 1.3
1.3 x 0.7 = 0.91 0.91 ÷ 1.3 = 0.7</td></tr>
<tr><td></td><td></td><td> 



14 x 36 = 504 504 ÷ 14 = 36
36 x 14 = 504 504 ÷ 36 = 14</td><td> 

$2\frac{1}{2} \times 1\frac{1}{3} = 2\frac{8}{6} = 3\frac{2}{6}$
$1\frac{1}{3} \times 2\frac{1}{2} = 2\frac{8}{6} = 3\frac{2}{6}$</td></tr>
</table>

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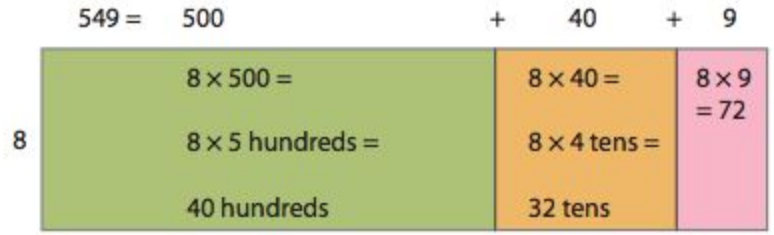
Grade 3 – Grade 5

Connection to Algorithms: Multiplication (3 x 1 digit)

Array/area drawing for 8 × 549



$$8 \times 549 = 8 \times (500 + 40 + 9)$$
$$= 8 \times 500 + 8 \times 40 + 8 \times 9$$

Method A:

Left to right showing the partial products

549 × 8

	thinking:
4000	8 × 5 hundreds
320	8 × 4 tens
72	8 × 9
4392	

Method B:

Right to left showing the partial products

549 × 8

	thinking:
72	8 × 9
320	8 × 4 tens
4000	8 × 5 hundreds
4392	

Method C:

Right to left recording the carries below

549 × 8

4 3 7
022
4392

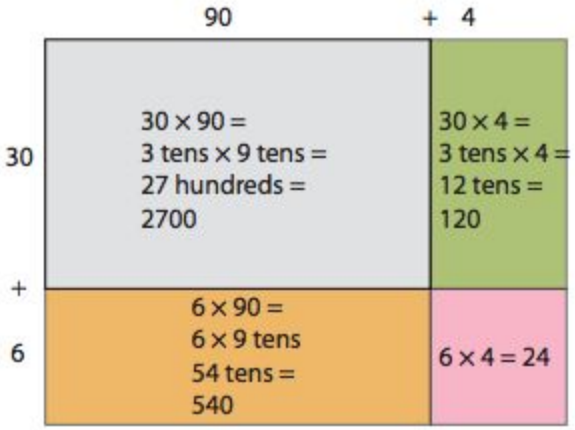
Method A proceeds from left to right, and the others from right to left. In Method C, the digits representing new units are written below the line rather than above 549, thus keeping the digits of the products close to each other, e.g., the 7 from 8x9=72 is written diagonally to the left of the 2 rather than above the 4 in 549.

Fuson, Karen C. and Beckmann, Sybilla. *Standard Algorithms in the Common Core State Standards.* NCSM Journal. Fall/Winter 2012-2013.
https://www.mathedleadership.org/docs/resources/journals/NCSMJournal_ST_Algorithms_Fuson_Beckmann.pdf

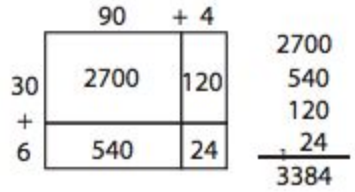
Visual Models:
Multiplication and Division
Grade 3 – Grade 5

Connection to Algorithms: Multiplication (2 x 2 digit)

Array/area drawing for 36 × 94

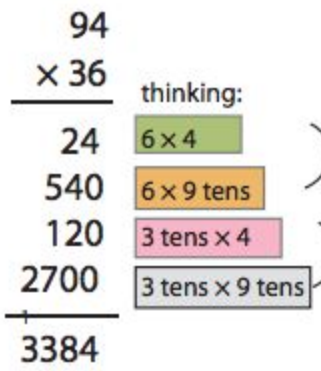


$36 \times 94 = (30 + 6) \times (90 + 4)$
$= 30 \times 90 + 30 \times 4 + 6 \times 90 + 6 \times 4$



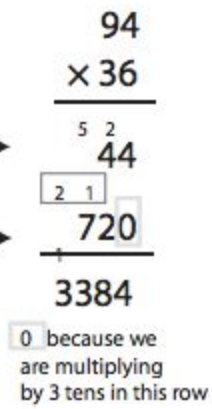
Method D:

Showing the partial products



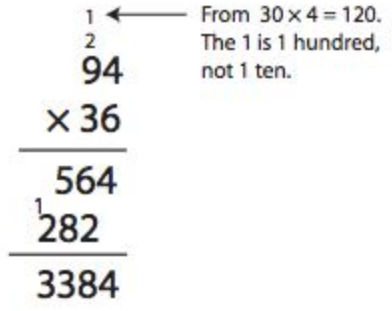
Method E:

Recording the carries below for correct place value placement



Method F:

A misleading abbreviated method



Method G: Helping Steps

$$\begin{array}{r} 94 = 90 + 4 \\ \times\ 36 = 30 + 6 \\ \hline 30 \times 90 = 2700 \\ 30 \times 4 = 120 \\ 6 \times 90 = 540 \\ 6 \times 4 = 24 \\ \hline 3384 \end{array}$$

Written Methods D and E are shown from right to left, but could go from left to right. In Method E, digits that represent newly composed tens and hundreds in the partial products are written below the line instead of above 94. This way, the 1 from 30x4=120 is placed correctly in the hundreds place and the digit 2 from 30x90=270 is placed correctly in the thousands place. If thse digits had been placed above 94, they would be in incorrect places (as in Method F). Note that the 0 in the ones place of the second line of method E is there because the whole line of digits is produced by multiplying by 30 (not 3).

Method G is a "helping step" version of Method D. By writing out the tens and the ones in each factor, students can see the number of zeros, and thus use the patterns involving tens and hundreds more easily. By writing the factors for each partial product, they could check on whether all partial products were included.

Fuson, Karen C. and Beckmann, Sybilla. *Standard Algorithms in the Common Core State Standards.* NCSM Journal. Fall/Winter 2012-2013.
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Connection to Algorithms: Division

Area/array drawing for 966 ÷ 7

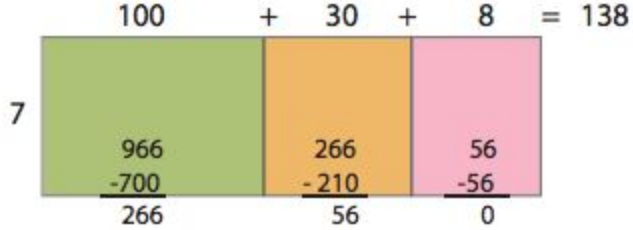
? hundreds + ? tens + ? ones

$$7\overline{)966}^{???}$$

Thinking: A rectangle has area 966 and one side of length 7. Find the unknown side length. Find hundreds first, then tens, then ones.

$$\begin{aligned} 966 &= 7\times100 + 7\times30 + 7\times8 \\ &= 7\times(100 + 30 + 8) \\ &= 7\times138 \end{aligned}$$

Method A:



$$\begin{array}{r} 8 \\ 30 \\ 100 \\ \hline 7\overline{)966} \\ -700 \\ \hline 266 \\ -210 \\ \hline 56 \\ -56 \\ \hline 0 \end{array} \quad 138$$

Method B:

$$\begin{array}{r} 138 \\ \hline 7\overline{)966} \\ -7 \\ \hline 26 \\ -21 \\ \hline 56 \\ -56 \\ \hline 0 \end{array}$$

Conceptual language for this method (all numbers below 966 are in square units):

Find the unknown length of the rectangle;
first find the hundreds, then the tens, then the ones.

The length gets 1 hundred (units); 2 hundreds (square units) remain.

2 hundreds + 6 tens = 26 tens (square units).

The length gets 3 tens (units); 5 tens (square units) remain.

5 tens + 6 ones = 56 ones (square units).

The length gets 8 ones; 0 remains.

The "bringing down" steps represent unbundling a remaining amount and combining it with the amount at the next lower place.

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